Abstract

The paper contains a proposed experiment for testing the gravitomagnetic effect on the propagation of light around a rotating mass. The idea is to use a rotating spherical laboratory-scale shell, around which two mutually orthogonal lightguides are wound acting as the arms of an interferometer. Numerical estimates show that time of flight differences between the equatorial and polar guides could be in the order of $\sim 10^{-20}$ s, actually detectable with sensitivity perfectly comparable with those expected in gravitational wave detection experiments.

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Testing gravitomagnetism on the Earth

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I. Introduction

The gravitomagnetism is the part of the gravitational field which displays a solenoidal character similar to that of the magnetic field.[1] It is embedded in general relativity, however its effects are usually much less relevant than those of the gravitoelectric (radial) part of the field. Gravitomagnetism is expected to influence the precession of orbiting gyroscopes (Lense-Thirring effect[2]), the synchronization of clocks in the field of rotating masses,[3],[4] the time of flight of light around spinning bodies.[5]

Actually in the field of the Earth the relevance of gravitomagnetic effects is extremely small and the only attempts to detect them have since been limited to the precession of the nodes of the orbits of LAGEOS satellites[6] and to the precession of gyroscopes carried by the space shuttle.[7]

Here we propose a ground based experiment exploiting the effect on the time of flight of light rays, induced by a rotating mass. Actually, as we shall see, the time of flight is influenced both by the very mass M of the central body and by its angular momentum density, expressed by the parameter $a = J/(Mc)$ i.e. times the ratio between the angular momentum and the product of the mass by the speed of light. However when considering the time of flight difference between an equatorial and a polar circular trajectory, it turns out, at the lowest significant order, to be proportional to a^2.

The actual value of a depends on the geometry of the source and its angular velocity. The highest results are obtained in thin spherical shells.

For the whole Earth a is in the order of 4 m; at the laboratory scale it is some orders of magnitude lower, however we shall show that the final value for the time difference is within the sensitivity range of interferometric techniques presently available and under consideration for gravitational waves detectors. The expected cost for the proposed on Earth experiment should in turn be much lower than those in space.

II. The time difference

In an axially symmetric stationary gravitational field the null interval is written:

$$0 = g_{tt}dt^2 + 2g_{t\phi}dtd\phi + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2$$

where the metric elements do not contain either t or ϕ. Considering a circular path the (coordinate) time of flight for an equatorial revolution ($\theta = \pi/2$) is[8]

$$T_e = 2\pi \frac{\mp g_{t\phi} + \sqrt{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}}{g_{tt}} \tag{1}$$

The $-$ sign stands for co-rotation, $+$ means counter-rotation.

Using Boyer-Lundquist coordinates in a Kerr metric (1) becomes

$$T_e = \frac{2\pi}{c^2} \frac{\mp \frac{2GMa}{cr} + c\sqrt{\left(\frac{2GMa}{c^2r}\right)^2 + \left(1 - \frac{2GM}{c^2r}\right)\left(r^2 + a^2 + \frac{2GMa^2}{c^2r}\right)}}{1 - \frac{2GM}{c^2r}} \tag{2}$$

The other configuration we are considering is a fixed azimuth polar circular trajectory. Now it is

$$T_p = \int_o^{2\pi} \sqrt{-\frac{g_{\theta\theta}}{g_{tt}}}d\theta$$

or explicitly

$$T_p = \frac{1}{c}\int_o^{2\pi} \sqrt{\frac{r^2 + a^2\cos^2\theta}{1 - \frac{2\frac{GM}{c^2}r}{r^2+a^2\cos^2\theta}}}d\theta \tag{3}$$

In a weak field and introducing the small parameters $\mu = \frac{2GM}{c^2r}$ and $\alpha = a/r$ (2) and (3) become

$$T_e = \frac{2\pi}{c}R\left(1 + \frac{1}{2}\alpha^2 + \frac{1}{2}\mu\right)$$

$$
\begin{aligned}
T_p &= \frac{R}{c}\int_0^{2\pi}\left(1+\frac{1}{2}\alpha^2\cos^2\theta+\frac{1}{2}\mu\right)d\theta \\
&= 2\pi\frac{R}{c}\left(1+\frac{1}{2}\mu+\frac{1}{4}\alpha^2\right)
\end{aligned}
$$

Finally we obtain

$$\Delta T = T_e - T_p = \frac{1}{2}\frac{\pi}{cR}a^2 \tag{4}$$

which, as can be seen, depends only on a^2 (the terms containing the mass mix it with a and are smaller).

III. Laboratory scale

For an homogeneous steadily rotating sphere it is

$$a_f = \frac{2}{5}\frac{R^2}{c}\Omega \tag{5}$$

where Ω is the angular speed of the sphere.

The value of a can be increased a bit considering instead of a sphere a hollow spherical thin shell. In that case one has

$$a_h = \frac{2}{3}\frac{R^2}{c}\Omega \tag{6}$$

Now however the mass, for the same external radius, is much lower than before. Actually

$$\frac{M_h}{M_f} = 3\frac{h}{R}$$

where h is the thickness of the shell ($h << R$).

Applying (5) to the Earth the result is

$$a_E = 3.9\ m \tag{7}$$

Other examples are Jupiter or the Sun[9]:

$$
\begin{aligned}
a_J &= 1.2\times 10^3\ m \\
a_S &= 3.0\times 10^3\ m
\end{aligned}
$$

In the laboratory one can of course expect much lower values. Let us consider a hollow sphere as the source of the effect. The a value is limited in practice by the strength of the wall of the shell. In fact the resulting centrifugal force on a hemisphere is

$$F_c = \pi \rho h \Omega^2 R^3$$

(ρ is the density of the material).

The corresponding average tension induced in the wall of the shell is

$$< \sigma >= \frac{\pi \rho h \Omega^2 R^3}{2\pi R h} = \frac{1}{2} \rho \Omega^2 R^2$$

The maximum stress is attained at the equator, being in the order (unidimensional stresses are assumed):

$$\sigma_m = \rho \Omega^2 R^2$$

If σ_m coincides with the allowable resistance of the material the maximal peripheral velocity is $v_m = \sqrt{\sigma_m / \rho}$. The attainable value of a_h can consequently be written in terms of the properties of the material: $a_h = \frac{2}{3} \frac{R}{c} \sqrt{\sigma_m / \rho}$.

Finally the time difference (4) becomes

$$\Delta T = \frac{2}{9} \frac{\pi}{c^3} \frac{R^2}{R_l} \frac{\sigma_m}{\rho} \tag{8}$$

Here R_l is the radius of the light's path (a little bit greater than R).

Considering composite materials σ_m can be as high as 2000 MPa, with a density $\rho \sim 1700$ kg/m^3.[10] These values lead to (assuming, just to fix ideas, $R = 1$ m)

$$\begin{aligned} a_h &= 2.4 \times 10^{-6} \ m \\ \Delta T &= 3 \times 10^{-20} \ s \end{aligned}$$

Using visible light the relative phase shift corresponding to the time of flight difference is in the order of 10^{-5}.

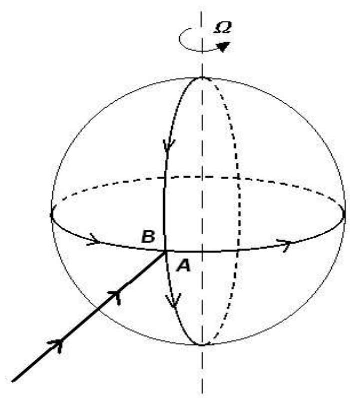

Figure 1: The hollow sphere rotates at the angular speed Ω. The two circular wave guides are fixed. A primary light beam is split in A to follow separately the equatorial and the polar trajectories, then the beam is recombined interfering in B.

IV. Proposing an actual experiment

A phase shift like the one computed in the previous section is of the same order of magnitude as the phase differences expected in gravitational waves interferometric detection experiments.[11] The advantage now could be the comparatively small size of the apparatus. The idea is to have as a gravitomagnetic source a spinning thin spherical shell; to fix a hypothesis it could have a 1 m radius and a wall thickness of 1 mm or less. Such object would weigh not more than 209 N and should rotate at a maximum angular speed of $\Omega_m \simeq 10^3$ rad/s. Two circular non rotating light guides should contour the sphere, one at the equator, the other through the poles, as in figure 1. A laser beam would be split at A, the two resulting secondary beams would be guided along the two circular paths and finally would be led to interfere at B.

The beam intensity relative change at the interference $\delta I/I$ is related to

the phase shift $\delta\Phi$ according to

$$\frac{\delta I}{I} = \frac{1}{2}\left(1 - \cos\delta\Phi\right)$$

Using the estimates of the previous section, this means

$$\frac{\delta I}{I} \sim 10^{-9}$$

It is not possible to extract such an intensity fluctuation if it is static. This means that we need modulating it in time; this result can be achieved periodically varying the angular speed of the sphere. In a sense we would be simulating the effect of a gravitational wave of very low frequency (reasonably fractions of a Hz).

V. Conclusion

We have shown that it is possible to realize a ground based experiment to reveal gravitomagnetic effects using a laboratory size rotating mass. In fact available materials (composite carbon fibers high resistance materials) and available technologies for detection of very small periodically varying intensity perturbations in a light signal do allow for the possibility to measure the time difference (4) and consequently the influence of the angular momentum density around a spinning body. Of course a lot of technical details need being clarified, but with no higher difficulty than the problems implied by interferometric detection of gravitational waves. The advantage of our proposal would be to have a cost presumably much lower than other experiments requiring satellites and space missions, revealing a small scale weak general relativistic effect.

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